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# ON THE FEASIBLE ENGINES OF THE VORTICAL FLOW ACTIVITY IN CLOSE BINARY STARS

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***Keywords:*** *Stars: Binary stars; Accretion; Instabilities: Vortices;*

***Abstract:*** *We present some possible mechanisms that provoke development of vortices in accreting flow in binary star systems. We use the standard configuration of close binary systems with the distance between the stars, comparable to their size. It includes: interacting flow between the components, mass transfer through the Lagrangian point, accretion disc around the compact object. Numerical gas dynamical calculations are applied on the base conditions of the baroclinic instability and Rossby wave instability. These two instabilities are considered to be the main engines for the vorticity production in the flow.*

# СЪСТОЯНИЯ НА НЕУСТОЙЧИВОСТИ КАТО МЕХАНИЗМИ ЗА РАЗВИТИЕ НА ВИХРОВИ СТРУКТУРИ В ТЕСНИ ДВОЙНИ ЗВЕЗДИ

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***Ключови думи:*** *Звезди: Двойни звезди; Акреция; Неустойчивости: Вихрови структури;*

***Резюме:*** *В тази статия са представени някои възможни механизми, които биха могли да предизвикат развитието на вихри в акреционното течение в двойни звездни системи. Използвана е стандартна конфигурация на тясна двойна система с разстояние между звездите, сравнимо с техните размери. Това също включва наличие на взаимодействащо течение между компонентите, трансфер на маса през точката на Лагранж, акреционен диск около компактния обект.*
*Приложени са числени газо-динамични симулации върху основните условия за възникване на бароклинна неустойчивост и неустойчивост на Розби вълни. Тези две неустойчивости са считани за основни действащи механизми при образуването на вихри в потока.*

## I. Introduction

In close binary stars, the most common processes are related to the flow interaction between the components. The incoming flow usually meets the matter around primary star and the resulting effect comes to the accretion disc structure formation.

This research is based on the aim of searching the mechanisms that enforce angular momentum transportation and this way to increase of accretion rate. A configuration of vortices in the regions with accreting flow is one of the most considered mechanisms as an efficient angular momentum transportation [2] in regions where the magneto-rotational instability [1] does not operate.

The simulations of Klahr and Bodenheimer in [8] show that non-magnetic turbulence can drive outward angular momentum transport and is maintained itself by the resulting accretion process.
There are many hydrodynamics studies in finding the way of vortices appear and their further behavior in the flow. According to the investigations of Klahr [8], baroclinic instabilities could arise in rotating fluids when there is an inclination between surfaces of constant density and the constant pressure [22]. In this case vorticity per unit surface density is not conserved (Kelvin's circulation theorem, see e.g. Pedlosky 1987 [19]), and vortices can be generated.

Vortices can be generated by a globally unstable radial entropy gradient [8] that may result in local outward transport of angular momentum. The first global (2D) simulation of an accretion disc allowing for the baroclinic instability results in large vortices form. These vortices are long-lived high-pressure anti-cyclones with an over-density by a factor of up to four.

Johnson and Gammie in [18] give a very realistic way of the initial vorticity generation. They have noted that the remaining vorticity can be generated from "finite-amplitude compressive perturbations". Shen et al. [21] examine the formation of 2D vortices starting from 2D turbulence in fully compressible simulations. Such a situation could be baroclinically unstable, and in fact that region is found to be associated with outward transport of angular momentum. Lyra in [14] talks about the SBI (Subcritical Baroclinic Instability), which is processing to keep up large-scale vortices in the presence of a radial entropy gradient and thermal diminution.

Three-dimensional, cylindrical numerical simulation shows a growth of strong and steady vortex in a differentially rotating disc. It appears in the result of the Rossby Wave Instability [10], [13], [16], which has been studied in various astrophysical contexts such as in protoplanetary discs or in the accretion disc of compact objects.

Lovelace et al. [13] and Li et al. [10] investigated the stability of a strong local entropy maximum in a thin Keplerian disk and found the situation to be unstable to the formation of Rossby waves, which transported angular momentum outward and ultimately formed vortices [11].

In this paper we suggest our interpretation and results of the conditions and mechanisms that could cause development of vortical – like structures in the accreting flow in binary star system.

## II. Engines that cause vorticity in the accretion flow

a) Disturbances in the flow and equations.

In the result of the mass transfer from the secondary to the primary star, the flow around the accreted object doesn't remain stable. It could be disturbed in the terms of velocity and density in time:

$V = v + u$ ; $\rho_q = \rho + \rho'$ ; $p = P + p'$ ; $\Psi_q = \Psi + \Psi'$

where $V, \rho_q, p, \Psi_q$ are the total quantities of velocity, density, pressure and vorticity respectively;

$v, \rho, P, \Psi$ are the time averaged values; $u, \rho', p', \Psi'$ are the perturbations in time. The exponential form of the perturbation terms [3] give influence on the Navier – Stokes equations (eqs. 1), (see also [3], Boneva & Filipov 2012: Appendix – this equation is expressed in cylindrical coordinates ($r, \varphi, z$) with the perturbed quantities) and on the vortical transport equation (eq. 2) [3].

(1) $$\frac{\partial v}{\partial t} + v.\nabla v = -\frac{1}{\rho}\nabla P - \Omega \times \left(\Omega \times r\right) - 2\Omega \times v - \nabla \Phi + \nu \nabla^2 v$$

The basic notations are: $\rho$ - is the mass density of the flow, $v$ - is the velocity of the flow; $P$ - is the pressure; $\nu$ - is the kinematics viscosity; $\Omega$ - is the angular velocity; $\Omega \times \left(\Omega \times r\right)$ is the centrifugal acceleration of the centrifugal force; and $2\Omega \times v$ is the Coriolis acceleration in the mean of the Coriolis force. In the current analysis $\rho \neq const$ and $\nu \neq 0$. $\Phi$ is the gravitational potential and it depends on the density distribution inside each of the star's component [5].

After the flow has once been perturbed, it could be transformed to the variety of unstable states, which might produce a development of vortical like structures throughout the whole disc. One way to explain the vortex formation is the operation of the vorticity equation (or its transformation as a vortical transport equation).

The vortical transport equation is related to the examination of transfer's mechanisms in the flow. This equation could be derived in the following commonly used way, how it has been done by Nauta, Lithwick, Godon in [7], [12], [17]. If the curl of Navier-Stokes equations is considered and using the expressions: $\Psi = \nabla \times v$ , which gives the vorticity in the flow; as well as:

$(v.\nabla)v = \nabla v^2/2 - v \times \Psi$ and $\left(\frac{\partial \Psi}{\partial t} + v.\nabla\right)\frac{1}{\rho} = \frac{\nabla \rho \times \nabla P}{\rho^3}$,

then the following expression, which we would call "the vortical transport equation" is obtained:

(2) $$\frac{\partial \Psi}{\partial t} + \Psi(\nabla . v) + (v . \nabla)\Psi = -\frac{\nabla p \times \nabla \rho}{\rho^2} + D\nabla^2 \Psi$$

Here $\Psi$ - is the vorticity; $D$ - is the diffusion coefficient (or matrix of the transport coefficient).
This equation expresses the relation between the transport coefficient, which takes part in the angular momentum transfer, evolution of the vorticity with time and the non-conserve relationship between density and pressure in the flow. Non-conservancy of specific vorticity by the each fluid element is observed in the right-hand side of Eq. (2), which gives a condition for baroclinicity in the flow [13] and enables the development of vortices in the accretion matter.

b) Baroclinic instability

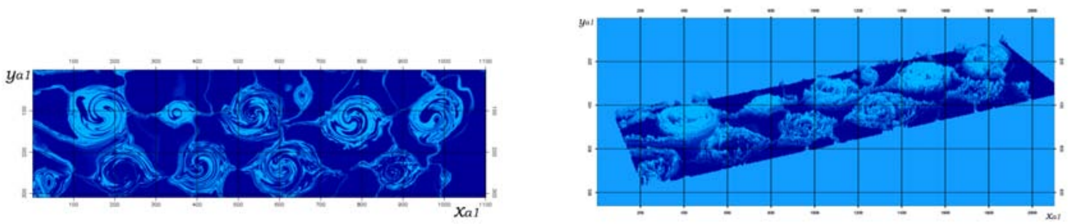
According to the baroclinicity conditions of Klahr and Bodenhiemer [8], misalignment of pressure gradient and density gradient is required wherever there is azimuthal density gradient. This misalignment acts as a source term for the generation of vorticity in the region of outer edge of the disc.

The baroclinicity of the general flow is given by the baroclinic term [9], [20]: $\nabla\rho(r,z,\varphi)\times\nabla p(r,z,\varphi)\neq 0$ . The importance for this instability is the non-axisymetric deviations from the mean state which can lead to the rise of the baroclinic term even in two dimensions: $\nabla\rho(r,\varphi)\times\nabla p(r,\varphi)\neq 0$ and vorticity can be generated. This instability, known as baroclinic [8], [9] and the corresponding baroclinic term is responsible for the vortex production, as it is the only source term in the vorticity equation. Following this formulation, we perform a computational analysis to reveal one possible way of the vorticity appearance, based on the vortical transport equation (Eq. 2), [8]. For the purpose, the box - frame model is created (see Boneva & Filipov, 2012) [3].

The introduced boundary conditions in this model are of Dirichlet- and Cauchy type: $r_v(1+n) = K(x,y) - \frac{\partial K}{\partial r_v}\frac{\partial}{\partial t}$, $r_{v0}(0)=0$ is the radius of the vortex; $K(x,y)$ is the boundary area of the equations activity: $\sim 10^8 m^2 \sim 6.68\times 10^{-10} AU$, for $x$ and $y$ respectively. We place the cylindrical coordinates $(r,\varphi,z)$ frame for the equations and quadratic $(x,y)$ set for the numerical scheme. We perform a series of runs with zero initial vorticity, but different from zero the initial turbulence values: $v(0)=v_0$, $\Psi_{r,\varphi}(t_0)=0$, $\rho(t_0)=\rho_0 \approx 2.5\times 10^{-6} kg/m^{-3}$, $t_0 \approx 1$, and $r_0 \approx 1$.

Results of the simulations show a vortex type growth in $r,\varphi$ plane of the disc zone. The box-frame values range from about $7.6\times 10^{-3} AU \ldots to \ldots 6.68\times 10^{-3} AU$ and from $7.6\times 10^{-6} AU \ldots to \ldots 6.68\times 10^{-6} AU$, corresponding to the above values of $x$ and $y$, referred to as $x_{a1}$ and $y_{a1}$.

After the numerical analysis, given in details in [3], the development of vortical-like patterns in the flow could be expressed in the next simulations:




(a) (b)

Fig. 1. Simulations of the vortical patterns production in the result of Baroclinic instability ([3], [4]). Figures illustrate the final stages of the development of vortical structures in the disc flow in 2D (fig. 1a) and 3D (fig.2b) view. The sizes of the pictures frames referred to the defined in boundary conditions values, ranged for $x_{a1}$ and $y_{a1}$. The light Blue and dark Blue colours (light and dark in a grey scale for the printed version) show the difference in density in the interacting flow layers. The density values are increasing from dark to the light zone.

Now we can summarize that to describe the vortex production we need: the vortex equation, which to be converted in the vortical transport equation; plus baroclinicity conditions as a source of the vortex.

The analysis on the construction of vortical transport equation, (see Boneva & Filipov 2012, Eq. 5) [3], shows that it is an expression of the whole disc's structure analog. It gives the relation between the angular momentum transport part (the last term in the right hand side) and describes the sources of the transport mechanism in sense of the vorticity function, and the patterns development.

c) Rossby wave instability

In an unmagnetized differentially rotating disc, the vortices can be generated by the operation of Rossby Wave Instability [11], [13], [15]. This instability is usually proceeds in areas with variations in the density values and density accumulation.

The instability occurs, where the local extremum in the radial profile of the quantity $\Pi(r)$ exists, related to the vorticity of the equilibrium flow. It is initially given by Li et al. in [10], later applied by Meheut et al. in [15], [16] and here takes the form:

$\Pi(r) = \Lambda(r)\dfrac{P}{\rho^{\gamma}}$ ; $\Lambda \approx \dfrac{\rho\Theta}{f^2} = \dfrac{\rho}{2\Psi}$; ( $f^2/\Theta$ is the vorticity $\Psi$ ); $f^2$ is the square of the epicyclic frequency, $\rho$ is the volume density, $\gamma$ the adiabatic index, $\Theta$ is the rotation frequency. Then we have: $\Pi(r) = \dfrac{\rho}{2\Psi}\dfrac{P}{\rho^{\gamma}}$.

One possibility for the extremum in $\Pi(r)$ is found to be near marginally stable orbit around the compact object, where the maximum of the dispersion relation of Rossby waves could be written here in the next form, following the expressions of Lovelace [13] and Meheut [15]:

(3) $$\Delta\omega = -\frac{f_\varphi c_S/\Theta}{1+f^2h^2}\left[(ln\,\Pi)' \pm \sqrt{[(ln\,\Pi)']^2 - \frac{1+f^2h^2}{L_S L_p}}\right]$$

where $L_s$ is the length scale of the entropy variation, $L_p$ is the length scale of the pressure variation; $\omega$ is the frequency mode; $c_S$ - the speed of sound; $f^2 = f_r^2 + f_\varphi^2 + f_z^2, h = c_S/\Theta$; $\Theta = v_\varphi / r$.
Following Eq.(3) the maximum occures for $(ln\,\Pi)' = 0$ and it is obtained to be:

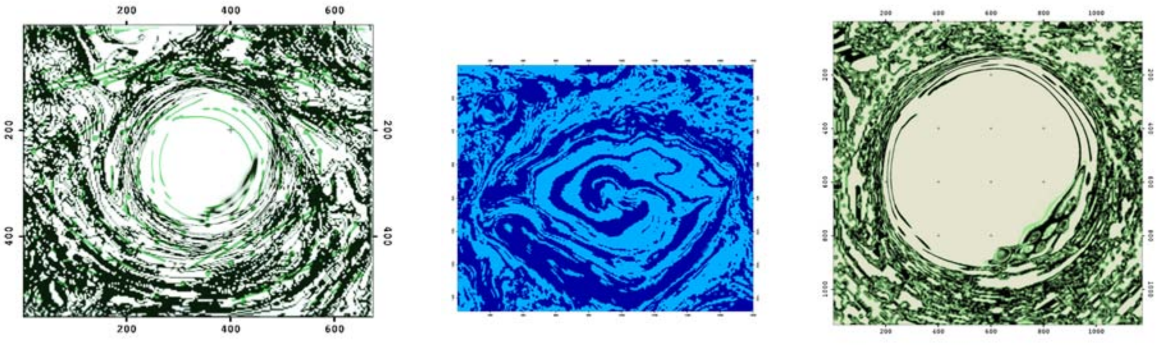
(4) $$\frac{max(\omega_i)}{\Theta} = -\frac{|f_\varphi| c_S^2/\Theta^2}{\left(1+f^2h^2\right)^{1/2}\left(L_S L_p\right)^{1/2}}$$

Then, its simplified form becomes:

(5) $$\omega_i(max) \approx \frac{|f_\varphi| h}{\left(1+f^2h^2\right)^{1/2}}\left(\frac{c_S}{v_k}\right)\left(\frac{r^2}{L_S L_p}\right)^{1/2}$$

Further, the equations 3-5 (Eqs. 3-5) are applied to investigate the flow states under the conditions and operation of the Rossby wave instability. For this calculation we employ 2D frame-scheme, and the same initially conditions as in the Section (II a).

Taking in account the whole disc's conditions and equations, the calculations are resulting in the next 3 figures. They give us one visualization of the vortices formation:




(a) (b) (c)

Fig. 2. Stages and snapshots pictures of the vorticity formation in the result of Rossby wave instability. Figure (a) illustrates the process of initial vortencity in the flow (in a $(r,\varphi)$ frame). It is presented as the velocity vectors (green lines) and density layers (dark and light thick lines) along the area, where Rossby vortex is generating. Figure (b) shows the single Rossby vortex, situated in the flow. The light and dark blue colors correspond to the different density layers. The simulation at fig. (c) presents the development of small vortex formations along to the inner part of the accretion disc.

The grids of the Figures 2 (a,b,c) correspond to the calculation frame scheme. Their sizes are not related to the boundary condition, given in the model explanation in Section (IIa) and [3].

## III. Conclusion

In the studied interaction parts of the binary star flow, the speed of the tidal flow is close to the speed of sound. Then, the incoming wave from the secondary to the primary star may cause some perturbations in quantities, which could change the dynamics of the gas flow. After applying the numerical methods via numerical code the result shows the presence of two-dimensional vortical patterns.

Vortices are usually local formations, but under the conditions assumed here, they could propagate globally throughout the disc. This type of vortices has often been observed in 2-D simulations of accretion discs. We have been started applying simulations in a 3-D calculating box - framed scheme, according to the model above, but still in a 2-D reference grid. The results in 3-D will be a part of the future work.

Vortices can be generated by two-dimensional instability as the Rossby wave instability or baroclinic instability, which have been investigated in recent years.

In depend on the dominant activity of the suggested in this paper vorticity generation engines, they could appear as single structures or in a group of several vortices.

In the cases, when the vortices are developed not as a single structure, they could be considered as a chain with tied to each other vortex formations. Further they can evolve as they start merge and form a larger vortex-like or other density structure. Or they could continue their movement throughout the whole disc until their decaying phase ends.

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